**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:-** Given-

µ = 45

σ= 8

Service manager starts work after the 10 min car drop off.

Therefore, X=50

We need to find P(Z<50).

Z- score- Z=(X−µ)/σ= (50-45)/8 = 0.625

>pnorm(0.625)

[1] 0.7340145

The service manager will take more than 50 mins to not meet his commitment .i,e**(Z>0.625) = 1- P(Z<50) => 1- 0.7340145=0.265985**

**Hence, Option B. 0.267 is the correct ans.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans:- False.** Because the average or mean is 38, and its normally distributed, which means the mode is also 38 and more of the employee age is 38 and the middle mostage is 38.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:- True.**

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:-**The Normal distribution has two parameters, themean, E(x)=µ,andthevariance,

Var(x)=σ2. µ and σ2 satisfy −∞ < µ < ∞, σ2> 0.

We write X ∼Normal(µ,σ2), or X ∼N(µ,σ2).

If X and Y are independent, and X ∼Normal(µ1, σ21), Y ∼ Normal(µ2, σ22),

then X + Y ∼Normal(µ1 + µ2, σ21 + σ22).

More generally, if X1,X2,...,Xn are independent, and

Xi ∼Normal(µi,σ2i ) for i = 1,...,n,

then aX1+a2X2+...+anXn∼Normal((a1µ1+...+anµn), (a2 1σ2 1+...+a2 nσ2 n)).

Here,

*X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables

**2*X1*~ *N*(2μ, 4σ2)**

*X*1 + *X*2 ~ *N*(μ+μ, σ2+σ2) i.e. ***X*1 + *X*2 ~ *N*(2μ, 2σ2)**

Distribution for *X1* has normal distribution with sharper tails. Whereas, *X*1 + *X*2 has higher peak as compared to *X1*.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans :** Given –

X ~ N(100,20²)

Mean(µ) = 100

Std. Deviation () = 20

Variance(²) = 20²

Step 1

We need to identify symmetric value for the standard normal distribution such that the area enclosed is 0.99. Here, the idea is to recognize that the excluded area is 0.005 in

each of the left and right tails. Thus we want to find the 0.5th and 99.5th percentiles.

As it’s a normal distribution , let mean = 0 ,standard deviation = 1,Two tailed

Pr[-x < X < +x]T and for central probability(in %) is 99. Hence , we get the 0.5th and

99.5th Percentiles values + or – 2.576.

Step 2

Using the standardization formula as your starting point , solve backwards for the corresponding 0.5th and 99.5th percentiles of a normal distribution with mean 100 and

standard deviation 20.

Z = (x-μ)/ says that x = + μ

Thus “a” = 0.5th percentile for X=20[-2.57] + 100 = 48.6

And “b” = 99.5th percentile for X=20[+2.57] +100 = 151.4

Hence, the ans is **D.48.5,151.5**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans :** Total profit = profit1 + profit 2 ~  N(μ1+ μ2,²² 2)

=N(5+7,32+42)

~N(12,52) in $

Central probability (in %) = 95%

Let mean=0, standard deviation =1,Two tailed pr[-x < X < +x]T

Hence, we get 2.5th and 97.5th percentiles value as + or – 1.96.

X = +µ

Thus “a” = 2.5th percentile for x = 5[-1.96] + 12 =2.2$=2.2\*45=99 Rupees

And “b” = 97.5th percentile for x = 5[+1.96] + 12 =21.8$= 21.8\*45=981 Rupees

**Hence , the range (centered on the mean) such that it contains 95% probability for the**

**annual profit of the company is [99 981]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans :** To find 5th percentile from the left side we can use the formula,

µ-1.5 => 540-(1.5\*225) =>202.5 million rupees.

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans :** This question concerns the original profit distribution.

For division1= Z score for a profit of zero: Z=(X-μ)/=> (0-5)/3 => -1.66=0.0485

Or pnorm(0,5,3)

[1] 0.04779035